On-board State Estimation for Planetary Aerobots

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Abstract

Oscillatory balloon systems with telerobotic capabilities are being studied to support future space exploration by probes which will move up and down in a planetary atmosphere, land and explore numerous surface sites. On-board state estimation, supported by a variety of sensing sources, will be essential to support key balloon functions such as path prediction and planning, descent and landing operations, and science instrument pointing. The relevant states include position and velocity plus attitude and angular velocity. The sources of sensing information include inertial, image, celestial, range and radiometric sensing. A primary focus of this paper is the incorporation of inertial sensor data into the estimator design

1 Introduction

Oscillatory balloon systems with telerobotic capabilities are being studied to support future space exploration by probes which will move up and down in a planetary atmosphere, land and explore numerous surface sites. Current mission studies have focussed on Mars and Venus as primary targets of interest [2]. On-board state estimation, supported by a variety of sensing sources, will be essential to support key balloon functions such as path prediction and planning, descent and landing operations, and science instrument pointing. The relevant states include position and velocity plus attitude and angular velocity. The sources of sensing information include inertial, image, celestial, range and radiometric sensing [1]. Figure 1 shows a typical dual-phase robotic balloon configuration [6].

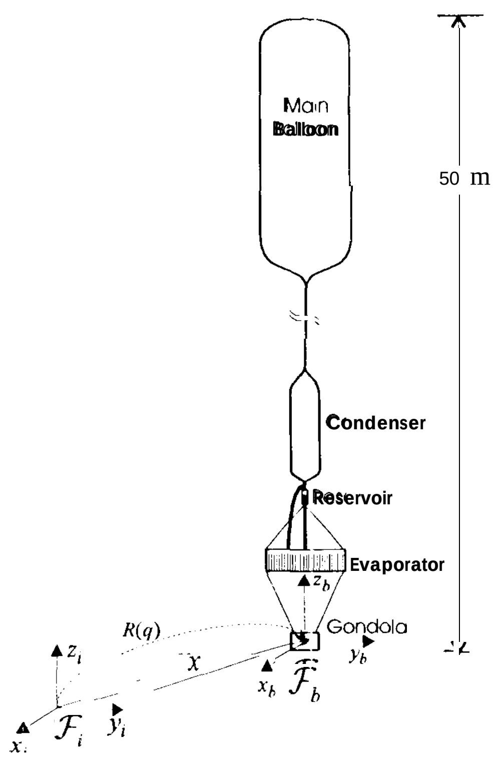

Figure 1: Robotic Balloon System

For Venus balloons, the position estimate is most important since there will be periods where the system is effectively blind with regard to seeing the surface (e.g., during daylight at float altitudes above a few tens of kilometers). In contrast, for Mars balloons the attitude estimate is more important since the rotation errors will directly feed into the celestial measurements.

A primary focus of this paper is the incorporation of inertial sensor data into the estimator design. Inertial sensing allows a robotic balloon system to determine or propagate its state without reference to any external sensing sources. Rate gyroscopes provide information on angular velocity and allow the estimator to propagate the attitude of the balloon system. Accelerometers provide information on true spatial accelerations as well as local gravimetric information by reacting to the direction of the local gravity field, that is, the local vertical.

The fusion of inertial sensor data for a balloon system, represents a new estimation regime that is different from that of a spacecraft or land rover. Unlike a spacecraft the balloon is not in free-fall; unlike a rover it is not in a quasi-static "rest" equilibrium. Instead the balloon system can be characterized as a platform with pendulum-like motion that combines aspects of free-fall as well as "rest" motion. The accelerometers sense a combination of the gravimetric vertical together with the non free-fall portions of the spatial acceleration.

In this paper, it is shown that the combination of rotation and translation information from the accelerometer measurement can be used to advantage for state estimation in a balloon system. Specifically, an estimator is developed which exploits the effect of the gravitational field on inertial sensors to make observable two out of three angular attitude variables. In Section 2 the model for a conventional inertial estimator is developed. Next in Section 3 a discussion motivates the exploitation of the gravitational field by the inertial sensors. In Section 4 a model is developed to exploit the gravitational field information. A nonlinear state estimation scheme is then designed in Section 5 based on the gravity projecting model. A simulation example is presented in Section 6. Conclusions are postponed until Section 7.

2 Sensor-Integrating Model

As a starting point, a basic rigid body dynamic model with forces and torques (cf. [4]), will be applied to the balloon problem as follows,

$$\dot{q} = \frac{1}{2}\,\Omega(\omega)q \tag{1a}$$

$$\dot{\omega} = I^{-1}(-\omega^{\times} I\omega + \tau) \tag{1b}$$

$$\dot{x} = v \tag{1c}$$

$$\dot{v} = f/m \tag{1d}$$

where,

$$\Omega(\omega) \stackrel{\Delta}{=} \begin{bmatrix} -\omega^{\times} & \omega \\ -\omega^{T} & 0 \end{bmatrix}; \quad \omega^{\times} \stackrel{\Delta}{=} \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \tag{2}$$

For simplicity all dynamics have been referenced to the system's center-of-mass (CM). Here m is the total system mass, $x \in R^3$ is the position of the system CM in Cartesian coordinates, $v \in R^3$ is the velocity, and $f \in R^3$ is the corresponding Cartesian force. Likewise for rotational variables, I is the system inertia matrix, $q \in R^4$ is the attitude quaternion, $\omega \in R^3$ is the angular rate, and $\tau \in R^3$ is the corresponding torque. The rotational variables q, ω, τ are resolved in the body frame, while the translation variables x, v, f are resolved in an inertial frame

Typical measurements for state estimation are given by 3-axis gyro and accelerometer measurements in the body frame,

$$\omega_m = \omega \tag{3a}$$

$$a_m = R(q)(f/m - g) \tag{3b}$$

For discussion purposes, the noises and biases in the sensors have been neglected. Here the gyro ω_m measures the angular rate ω in the body frame, and the accelerometer measures the inertial accelerations due specific (i.e., non-gravitational) forces, projected into the body frame. The matrix $R(q) \in R^{3\times 3}$ denotes the unique direction-cosine matrix corresponding to the quaternion q, and g is the gravity vector given by,

$$g = [0, 0, -g_0]^T \tag{4}$$

It is seen that g is directed downward in the inertial frame. As seen in (3b), the accelerometer signal a_m is sensitive the *specific force* $f - gm$ rather than the total force f on the body. The measurement equations can be rearranged to become,

$$\omega = \omega_m \tag{5a}$$

$$f/m = R^T(q)a_m + g \tag{5b}$$

According to a standard approach, the measurement equations 6 are *substituted* into the dynamics equations 1 to give,

$$\dot{q} = \frac{1}{2}\,\Omega(\omega_m)q \tag{6a}$$

$$\dot{x} = v \tag{6b}$$

$$\dot{v} = R^T(q)a_m + g \tag{6c}$$

Because the measurements have been substituted into the state equations, this formulation is denoted as the *sensor-integrating* estimator. It is emphasized that the rotational dynamics equation (1b) has been removed, and hence the torques and inertia properties of the system do not have to be known. Likewise, the acceleration measurement is used to replace the force in the dynamics equation (6c), so that only the gravity component g must be known. With initial conditions specified as q_0, x_0, v_0, (6) can be integrated numerically to give the state estimate at any time t.

It can be seen that any measurement noises in ω_m and a_m will add to the right hand side of (6) and enter as *process noise*. Hence, a nonlinear state estimation scheme based on this formulation effectively has no measurements and simply propagates a state equation with process noise. Without additional information for position/angle, the covariance of all components of the state estimate will inevitably degrade with time.

Although the above approach does not include additional complexities needed to deal with sensor biases, noises, frame transformations, coordinate transformations, etc., it is essentially the method used to propagate the state in most present-day inertial navigation schemes.

The main contribution of the present paper is to redesign the state estimator to make two out of three angular position variables observable. This may seem remarkable at first since one is effectively integrating accelerations and rates to get positions. However, the extra information comes from the known gravity vector g whose projection into the body frame is measured by the accelerometers. This is discussed in the next section.

3 A Thought Experiment

Consider a thought experiment where a 3-axis accelerometer package is sitting motionless in a gravitational field so that the net force acting on the package is $f = 0$. Then from (3b) it is seen that the accelerometer measures,

$$a_m = -R(q)g \tag{7}$$

Since g is known and a_m is measured, this clearly gives information about the attitude quaternion q. In particular, a_m measures the projection of the gravity vector g into the body coordinates which ensures persistent information on two of the three attitude angles. The advantage of this configuration is that with only a single additional angular measurement (such as an inexpensive sun sensor) the full 3-axis attitude q of the robotic balloon becomes observable.

There are several difficulties encountered when trying to exploit this approach

1 *Force Modeling*: Our simple thought experiment is an oversimplification because the force vector $f = 0$ is perfectly known. In a state estimator which exploits the projected g information, there will be difficulty distinguishing f/m from g in the accelerometer measurement (3b). This means that forces on the balloon must be carefully modeled and introduced as additional states in the estimator.

2 *Torque Modeling*: If our thought experiment was made more complicated by putting the accelerometer package in a rotating body displaced from the CM by vector ℓ, then it would measure the more complicated quantity (cf., Appendix A, letting the CM play the role of the pivot point),

$$a_m = R(q)(f/m - g) - \ell^\times \dot{\omega} + \omega^\times \omega^\times \ell \tag{8}$$

Due to the coupling with rotational acceleration $\dot{\omega}$ the torques must also be modeled and introduced as additional states in the estimator.

3 *State Dependent Observability*: If the accelerometer package is in free-fall, the instrument nominally measures $a_m = 0$ and there is no information about the projected g vector. Hence, the quality of the angular information will be state dependent.

4 Gravity-Projecting Model

In this section, an alternative to the sensor integrating estimator is given to take advantage of the additional projected g information. As pointed out earlier, this requires explicit modeling of forces and torques. To this end, the simple rigid body model of the previous section will be replaced in the present section by the rigid pendulum model shown in Figure 2. The point about which the pendulum pivots is defined as the *pivot point*. It is seen in Figure 2 that the body frame $\mathcal{F}_b$ is displaced below the pivot point by a vector L (which will be written in body coordinates as ℓ). Since all rotation takes place about the pivot point, it replaces the CM used in the previous section for the rigid body without pendulum dynamics. Earlier, the CM was taken to coincide with the body frame, which simplified the dynamics. However, in the present model, it is not possible to have the the body frame coincide with the pivot point since they must be separated to create the pendulum dynamics. The derivation of the accelerometer measurement equation in the Appendix is still applicable to this more complex pendulum arrangement when one associates the quantity L with the offset vector from the pivot point to the body frame.

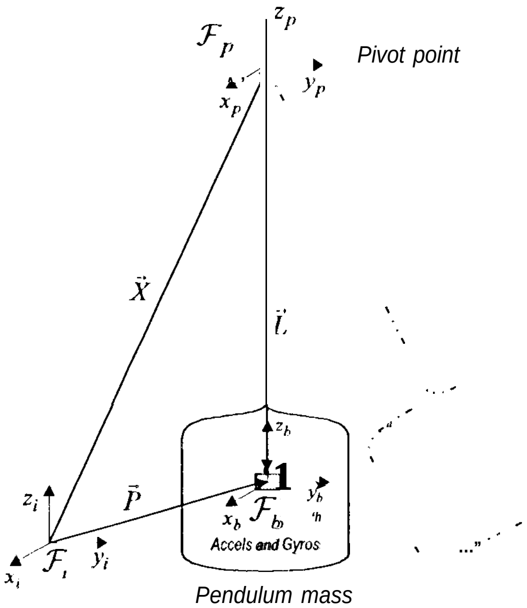

Figure 2: Pendulum Model for Balloon System

The state and measurement equations will first be given and then explained. The state equation is given as,

$$\begin{bmatrix} \dot{q} \\ \dot{\omega} \\ \dot{\tau} \\ \dot{x} \\ \dot{v} \\ \dot{f} \\ \dot{b} \end{bmatrix} = \begin{bmatrix} \frac{1}{2}\Omega(\omega_m)q \\ \Psi(\omega, q, \tau) \\ -A_\tau \tau \\ v \\ f/m \\ -A_f f \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ n_\tau \\ 0 \\ 0 \\ n_f \\ n_b \end{bmatrix} \tag{9}$$

where,

$$\Psi(\omega, q, \tau) \triangleq \begin{bmatrix} -2\zeta_x\lambda_x & 0 & 0 \\ 0 & -2\zeta_y\lambda_y & 0 \\ 0 & 0 & -2\zeta_z\lambda_z \end{bmatrix} \omega + \begin{bmatrix} \lambda_x^2 & 0 & 0 \\ 0 & \lambda_y^2 & 0 \\ 0 & 0 & \lambda_z^2 \end{bmatrix} \phi(q) + R(q) I_b^{-1} \tau \tag{10}$$

The measurement equation is given as,

$$\begin{bmatrix} \omega_m \\ a_m \end{bmatrix} = \begin{bmatrix} \omega + b \\ R(q)(f/m - g) - \ell^\times \Psi(\omega, q, \tau) + \omega^\times \omega^\times \ell \end{bmatrix} + \begin{bmatrix} n_g \\ n_a \end{bmatrix} \tag{11}$$

In (9) the forces f and torques τ have been modeled as first-order lowpass processes. This is to capture the effect of the wind and atmospheric effects on the balloon motion.

The dynamics of Ψ in (9) is introduced to model the "swaying" of the gondola, which behaves as a pendulum with resonance frequencies $\lambda_x, \lambda_y, \lambda_z$ about each of the three body axes. It is assumed that the parameters λ_x, λ_y, λ_z are known, since they can be characterized beforehand from the physics of the balloon. The rotations about the horizontal axes of the balloon will be modeled as simple pendula of length ℓ_b in a gravitational field (i. e., $\lambda_x \simeq \sqrt{g/\ell_b}$, $\lambda_y \simeq \sqrt{g/\ell_b}$, cf [5]). For simplicity one can take ℓ_b to be the length of the balloon system from top to bottom. (cf Figure 1) The rotation about the vertical axis of the balloon will be modeled with a very soft spring (i.e., $\lambda_z \simeq 0$) since there is no physical restoring force mechanism for this axis. The inertial scaling matrix I_b is correspondingly chosen to give a consistent scaling from torque to rotational acceleration in the pendulum model. Alternatively one can develop more detailed models for the effect of torques generated by the potential well associated with the gravitational field. But here the emphasis is on the use of conceptually simpler models which capture the essential qualitative behavior of the system.

The quantity $\phi(q)$ represents an incremental angular variable in local body coordinates which effectively is given pendulum dynamics. A number of modeling choices are available based on characterizations of attitude by Euler angles. Here the simplest small-deflection model has been used:

$$\phi(q) = \theta(q)u(q) \tag{12}$$

The angle/axis characterization, $\theta(q) \in \mathcal{R}^1$ and unit vector $u(q) \in \mathcal{R}^3$, provide a valid angular model for small (linear) deflections off the local vertical [4].

However, this simple model can also be justified qualitatively for somewhat larger deflections.

The bias b on the gyro is now modeled with an additional state in Equation 9 which integrates noise n_b. The effect of the offset vector ℓ which displaces the accelerometer package from the pivot point is also included, using the estimate of $\dot{\omega}$ now provided by Ψ.

Certain a-priori information and constraints have been carefully built into the estimator formulation to ensure that the projected g vector information is correctly utilized. These modeling issues are briefly outlined below.

1. The first-order lowpass noise process for f models the total external force (including gravity) on the balloon as zero-mean. In this manner, the desired $f = 0$ condition of the thought experiment is essentially being enforced in an average statistical sense. In the absence of external forces, this first-order model (assuming $A_f > 0$) ensures a contraction of the f estimate to zero, which resolves the $f - gm$ ambiguity and allows correct use of the projected g vector information.

2. The damping parameter $\zeta > 0$ in the pendulum model dampens the local angular coordinate $\phi(q)$ to zero in the absence of torques, ensuring that the preferred angular position for the balloon is vertical.

3. The first-order lowpass noise process for τ models the torques on the balloon as zero-mean. As with the forces, the condition $\tau = 0$ is being enforced in an average statistical sense. When coupled with the damped pendulum dynamics, this ensures that the balloon will have random motion inside a "potential well" with a preferred vertical position. In the absence of any external torques this first-order model (assuming $A_\tau > 0$), ensures contraction of the τ estimate to zero, while the damping parameter $\zeta > 0$ ensures that the q estimate will attain a vertical attitude.

4. The first-order lowpass noise statistics for the force and torque are specified completely by the parameters A_f, A_τ and variances of the corresponding noises n_f, n_τ. In practice, these parameters can be determined by approximating the power spectrum of the wind disturbances by first-order lowpass processes.

5 Nonlinear State Estimation

In this section, a nonlinear state estimation scheme is developed based on the gravity projecting model of Section 4. The basic idea is to define an intermediate local model to which the standard continuous-discrete Extended Kalman Filter (EKF) equations can be applied [3].

Let the quaternion q be given in terms of angle/axis variables (θ, u), where $\theta \in \mathcal{R}^1$ and unit vector $u \in \mathcal{R}^3$, as [4]:

$$q = \begin{bmatrix} \epsilon \\ \eta \end{bmatrix} \quad (13a)$$

$$\epsilon = u \sin(\theta/2) \quad (13b)$$

$$\eta = \cos(\theta/2) \quad (13c)$$

Because the normalization constraint $q^T q = 1$ is not explicitly enforced in the estimator design, the quaternion q acts as an over-parameterized representation of the 3-axis attitude. In order to avoid the redundant state, a local angular variable $\theta \in \mathcal{R}^3$ is defined by a local linearization at the beginning of each propagation step. After the completion of the corresponding update step, the local angular variable is absorbed into the quaternion from the previous linearization. The process is continued with subsequent linearizations. The corresponding covariance terms are maintained in terms of the local angular variable, which characterizes a ball of uncertainty about the attitude estimate.

For some $t = t_0$ and $q(t_0) = q_0$, a local angular variable θ is defined by the change of variables:

$$q(t) = \begin{bmatrix} \theta(t)/2 \\ 1 \end{bmatrix} q_0 \quad (14)$$

which can be solved to give:

$$\theta(t) = 2\epsilon\{q\, q_0^*\} \quad (15)$$

where $\epsilon\{\cdot\}$ is used to indicate the first three arguments of the quaternion as given in (1a). Then from (1a) and (2) the linearized equation for θ becomes:

$$\dot{\theta} = \omega - \frac{1}{2}\omega^\times \theta \quad (16)$$

Note that this parameterization is linear in θ but preserves the nonlinear dependence of ω for this accuracy in θ. With this substitution for q in (9), let the Kalman filter state be denoted by

$$X = (\theta, \omega, \tau, x, v, f, b)^T \quad (17)$$

Then using (14) to resolve all q-dependence gives, from (9)-(10),(16) (i.e., substitute (14) where all q-dependence appears below), the new local state equation for X:

$$\begin{bmatrix} \dot{\theta} \\ \dot{\omega} \\ \dot{\tau} \\ \dot{r} \\ \dot{v} \\ \dot{f} \\ \dot{b} \end{bmatrix} = \begin{bmatrix} \omega - \frac{1}{2}\omega^{\times}\theta \\ \Psi(\omega, q, \tau) \\ -A_{\tau}\tau \\ v \\ f/m \\ -A_f f \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ n_{\tau} \\ 0 \\ 0 \\ n_f \\ n_b \end{bmatrix} \quad 8$$

where,

$$\Psi(\omega, q, \tau \stackrel{\Delta}{=}$$

$$\begin{bmatrix} -2\zeta_x\lambda_x & 0 & 0 \\ 0 & -2\zeta_y\lambda_y & 0 \\ 0 & 0 & 2\zeta_z\lambda_z \end{bmatrix} \omega +$$

$$\begin{bmatrix} \lambda_x^2 & 0 & 0 \\ 0 & \lambda_y^2 & 0 \\ 0 & 0 & \lambda_z^2 \end{bmatrix} \phi(q) + R(q \; I_b^{-1}\tau \quad 9$$

As the rement equation is given as

$$\begin{bmatrix} \omega_m \\ a_m \end{bmatrix} =$$

$$\begin{bmatrix} \omega + b \\ R(q)(f/m \quad g) - \ell^{\times}\Psi(\omega, q \; \tau \; + \omega^{\times}\omega^{\times}\ell \end{bmatrix} +$$

$$\begin{bmatrix} n_g \\ n_a \end{bmatrix} \quad (20$$

This is a locally valid nonlinear system which avoids difficulties associated with the redundancy of a quaternion representation of attitude. However, it is nonlinear and remains to be *further* linearized in the usual EKF sense to realize the propagation and update stages of the filter.

The overall nonlinear state estimation scheme for propagating the state X, covariance P, and associated quaternion state q can be outlined as follows:

For given $X_{k-1}(+)$, $P_{k-1}(+)$, $q_{k-1}(+)$ reparameterize in terms of the local quaternion state $q_0 = q_{k-1}(+)$ to get state and measurement equations of the form (18)-(20)

2 Implement standard continuous-discrete EKF algorithm (cf. [3], p. 188) on equations (18)-(20).

(i) *Propagate* state and covariance from t_k o t_k:

$$X_{k-1}(+) \rightarrow X_k(-) \quad (21a)$$

$$P_{k-1}(+ \rightarrow P_k(-) \quad (21b)$$

(ii) *Update* state and covariance at t_k:

$$X_k(-) \rightarrow X_k(+ \quad (22a$$

$$P_k(- \rightarrow P_k + \quad (22b)$$

3. *Update* associated quaternion state and re initialize local angular variable at t_k:

$$q_k(+) = \begin{bmatrix} \theta_k +)/2 \\ 1 \end{bmatrix} q_{k-} +) \quad (23a)$$

$$\theta_k(+) = 0 \quad (23b)$$

4 Proceed to next stage of propagation and updating ($k \leftarrow k+1$)

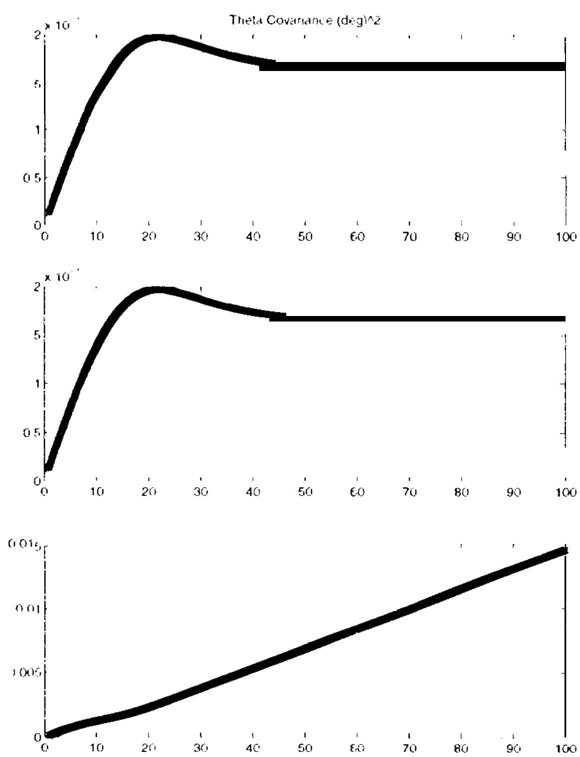

Figure 3: Covariance Analysis for Loca Angular Variable

6 Simulation Example

The filter design presented in Section 4 has been implemented in Matlab code. A simple example presented here illustrates the observability of two out of three axes of rotation. Modeling parameters in (9)-(11) were chosen to be consistent with proposed mission studies for robotic balloons [6]: $m = 15\,Kg$, $\lambda_x = \lambda_y = 0.44$, $\lambda_z = 0.$, $\zeta_x = \zeta_y = \zeta_z = 0.01$, $A_\tau = 100.\,s^{-1}$, $A_f = 100.\,s^{-1}$. Noise covariance parameters were also chosen to be consistent with the application with the bias term was zeroed out: $\sigma(n_\tau) = 1.0\,Kg\,ms^{-3}$, $\sigma(n_f) = 1.0\,Kg\,ms^{-3}$, $\sigma(n_b) = 0.0\,s^{-2}$, $\sigma(n_g) = 2.0 \times 10^{-4}\,s^{-1}$, $\sigma(n_a) = 1.4 \times 10^{-2}\,ms^{-2}$.

The simulation was derived from a plant model with the balloon system pointing vertically downward (i.e. $q = (0,0,0,1)^T$) but with a very poor initial condition

$$q_0 = \begin{bmatrix} \sin(\theta_0/2)u_0 \\ \cos(\theta_0/2) \end{bmatrix} \tag{24a}$$

$$u_0 = (1/\sqrt{2}, 1/\sqrt{2}, 0)^T \tag{24b}$$

$$\theta_0 = \pi/2 \tag{24c}$$

The covariance plot in Figure 3 illustrates bounded covariances for the x and y body axes while the angular covariance about the z body axis grows linearly. This validates that the estimator design works as intended, providing the desired observability of 2 angular rotations without the explicit use of attitude measurements.

7 Conclusion

A nonlinear state estimator has been developed for a robotic balloon. The state estimator integrates inertial sensor data (i.e., tri-axis accelerometers and gyros) to provide estimates of both attitude and position. A qualitative model for the pendulum-like balloon motion was developed to capture the kinematic behavior while avoiding unnecessarily complex dynamical models.

A main result of the new formulation is that two out of three local degrees of freedom in the attitude estimate become observable through the accelerometer measurements of the local gravitational field. By contrast, a more conventional estimator design based on direct integration of the inertial measurements leads to unbounded growth in the entire attitude covariance due to the integrated effect of the rate-gyro random-walk biases. An important advantage of the new estimator design is that only one additional attitude measurement is required to make the attitude fully observable. Such an additional measurement can be economically obtained using a measurement of the line-of-sight to the sun, a moon of the planet, the horizon, or a star.

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Appendix A Accelerometer Measurement Equation

The accelerometer measurement equation will be briefly derived. Define X as the coordinate free vector from an inertial reference point O to the vehicle pivot point L as the coordinate free vector from the vehicle pivot point to the accel package, and P as the coordinate free vector from the inertial reference point O to the accel package, i.e., $P = X + L$. The solid overdot "$\cdot$" will denote differentiation in the inertial frame while the overcircle "o" will denote differentiation in the vehicle body frame. Let Ω denote the (coordinate-free) angular velocity of the body. Differentiating $P = X + L$ in the inertial frame, using the well-known differentiation rule $\dot{L} = \Omega \times L + \overset{o}{L}$ (cf. [4]) gives,

$$\begin{aligned} \dot{P} &= \dot{X} + \Omega \times L + \overset{o}{L} \\ &= \dot{X} + \Omega \times L \end{aligned} \tag{25}$$

Here, the second equality follows from the fact that $\overset{o}{L} = 0$ since the accelerometer package is rigidly clamped (by assumption) to the vehicle body. Differentiating one more time in the inertial frame gives,

$$\begin{aligned} \ddot{P} &= \ddot{X} + \Omega \times (\Omega \times L) + \overset{o}{\Omega} \times L + \Omega \times \overset{o}{L} \\ &= \ddot{X} + \Omega \times (\Omega \times L) + \overset{o}{\Omega} \times L \end{aligned} \tag{26}$$

Now, let P and X be resolved in the inertial frame as p and x respectively, and let L be resolved in the body frame as ℓ. The angular rate Ω is also resolved in the body frame as ω. Then, the inertial acceleration resolved in the inertial frame is given by,

$$\ddot{p} = \ddot{x} + R^T(q)\left(\omega^\times \omega^\times \ell + \dot{\omega}^\times \ell\right) \tag{27}$$

The linear acceleration $\ddot{x}$ of the vehicle pivot point is determined by the total force f as,

$$\ddot{x} = f/m = g + (f/m - g) \tag{28}$$

Since the accelerometer is only sensitive to the specific force contribution $(f - gm)$, (28) is substituted into (27) and rearranged to give,

$$\ddot{p} - g = f/m - g + R^{T}(q)\,(\omega^{\times}\omega^{\times}\ell + \dot{\omega}^{\times}\ell) \tag{29}$$

Since the accelerometer is in the vehicle frame, we multiply on the left by $R(q)$ to give the accelerometer measurement a_m (without noise),

$$\begin{aligned} a_m &= R(q)(\ddot{p} - g) \\ &= R(q)(f/m - g) + \omega^{\times}\omega^{\times}\ell + \dot{\omega}^{\times}\ell \end{aligned} \tag{30}$$

Then (30) is the desired accelerometer measurement equation.

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